Revising Inconsistent Concepts

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0. Introduction

We aim to investigate the question of when it is reasonable to replace an inconsistent concept. By 'replace' we do not mean 'eliminate'. Instead, we are interested in the question of when it makes sense to introduce a new concept or concepts that are designed to fill at least some of the roles played by the concept discovered to be inconsistent. It might turn out that the concept in question is still used in certain situations even by those who recognize that it is inconsistent.

The main application of our inquiry is to the concept of truth and the so called inconsistency approaches to the paradoxes that affect truth (e.g., the liar, Curry, and Yablo). These approaches entail that truth is an inconsistent concept and that the paradoxes are symptoms of this inconsistency. Our question is: if truth is an inconsistent concept, then does it need to be replaced? Or more generally, when is the cure worse than the disease?[1]

1. Inconsistent Concepts

The first order of business is to say a bit about inconsistent concepts. The standard line is that a concept is inconsistent iff it has inconsistent constitutive principles. A major issue of contention among inconsistency theorists is how to understand constitutive principles. Many think that there is some connection between constitutive principles and concept possession. An old view of concept

[1] We borrow this medical analogy from Chihara (1979: 618) who writes: "So, in the end, it may be wiser to live with the illness than to undergo the kind of surgery needed to remove all paradox-producing elements."

possession has it that one possesses a concept iff one accepts that concept's constitutive principles. Here, constitutive principles are just thought to be analytic truths. Of course, this account is useless for anyone who wants a consistent theory of an inconsistent concept. For a set of analytic truths to be inconsistent, there would have to be sentences that are both true and not true, which anyone but a dialetheist would find unacceptable.

Instead, we ought to try another account of constitutive principles that does not take them to be factive. Matti Eklund thinks that a concept's constitutive principles are claims that anyone who possesses that concept is disposed to accept.[2] However, once one finds out that the concept in question is inconsistent, any rational person is going to reject one or more of its constitutive principles to avoid the contradiction. Of course, it does not make sense to think that once one discovers that a concept one possesses is inconsistent and one makes the requisite changes in the claims one is disposed to accept, one thereby loses possession of the concept. Thus, the 'disposed to accept' account seems unlikely to work well.

Instead, one might follow Douglass Patterson in thinking of the relation between a concept possessor and its constitutive principles as something like the Chomskian notion of cognizing, which is a subpersonal attitude that in the paradigm case obtains between a native speaker of a language and the grammar for that language.[3] The cognizer feels primitively compelled to accept that which is cognized, in much the same way that the lines in the Muller-Lyer illusion seem to be different lengths even after they have been measured. The problem with this proposal is that it does not respect the phenomenology of a person who has discovered that one of her concepts is inconsistent. Once one recognizes such a thing, one is no longer compelled to accept the inconsistent principles. Evidence for this is that those who find out a concept is inconsistent are not

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[2] Eklund (2002).
[3] Patterson (2006).

primitively compelled to accept a contradiction. Indeed, it seems that the compulsion goes the other way—the drive to reject contradictions leads one to think that the concept in question must be defective in some way.

In other work, one of us has defined 'constitutive' in terms of entitlements and scorekeeping.[4] For an interlocutor to say that a sentence p is constitutive of a word w in p is a scorekeeping commitment of the following sort: if a speaker rejects p when using w, then the interlocutor will think hard about whether w, for this person, means what the interlocutor means by it. The interlocutor will not just keep score transparently—without thinking about the meanings of the speaker's words. On this account, any person who possesses the concept expressed by w is entitled to p, unless there emerges evidence that w expresses a defective concept. In other words, rejecting a constitutive principle is an interpretive red flag; it is a defeasible indicator that one is not interpreting another properly. Although this scorekeeping notion of constitutivity would be fine for our purposes, it does bring with it a commitment to some kind of scorekeeping pragmatics, which although widely accepted by linguists who work on pragmatics, might be unacceptable for some readers.

One might consider developments in the literature on analyticity instead as a source of insight into constitutive principles. Since Quine's attacks on the analytic / synthetic distinction, there have been many attempts to refurbish the notion of analyticity.[5] The vast majority of them ential that analyticity is factive. Among them are proposals by Paul Boghossian, Gillian Russell, and Juhl and Loomis.[6] One recent account by David Chalmers is factive as well, but the factivity is tacked on to the primary account, which need not entail factivity. Chalmers calls his notion discursive analyticity. Here is his definition:

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[4] Scharp (2007, 2013).
[5] Quine (1951).
[6] Boghossian (1997), Russell (2008), Juhl and Loomis (2009).

(1) a sentence S is *dialectically analytic* iff S is true and any dispute over S involving at least one competent user of S is broadly verbal.[7]

Obviously, this won't do HERE since dialectical analyticity as defined is factive. Instead, we might prune away the first conjunct of the definiens and rename the definiendum to arrive at:

(2) a sentence S is *dialectically constitutive* iff any dispute over S involving at least one competent user of S is broadly verbal.

The following definition of 'broadly verbal dispute' is needed to understand the above definitions:

(3) A dispute over S is *broadly verbal* iff for some expression T in S, the parties disagree about the meaning of T, and the dispute over S arises wholly in virtue of this disagreement regarding T.[8]

Those with Quinean sympathies will probably be unhappy with Chalmers' appeal to meanings in this definition, despite the fact that Chalmers claims his notion of analyticity would be amenable to the Quinean.[9]

Instead, we might try something a bit more non-committal like John Burgess' pragmatic notion of analyticity. He illustrates his view by considering the law of excluded middle as an example in the following passage:

> My proposal is that the law should be regarded as 'basic', as 'part of the meaning or concept attached to the term', when in case of disagreement over the law, it would be helpful for the minority or perhaps even both sides to stop using the term, or at least to attach some distinguishing modifier to it. Such basic statements would then count as analytic, as would their logical consequences, at least in contexts where, in contrast with the examples above, there is no disagreement over logic. This proposal makes the notion of analyticity vague, a matter of degree, and relative to interests and purposes: just as vague, just as much a matter of degree, and just as relative to interests and purposes, as 'helpful'. But the notion, if vague, and a matter of degree, and relative, is also pragmatic, and certainly involves no positing of unobservable psycholinguistic entities, and for these reasons seems within the bounds of what a Quinean could accept.[10]

Burgess suggests the following definition:

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[7] Chalmers (2011: 557).
[8] Chalmers (2011: 522).
[9] Chalmers (2011: 561-2).
[10] Burgess (2004: 54).

(4) a sentence s is pragmatically analytic for a word w occurring in it iff in a disagreement over s, it would be helpful for at least one party to stop using w unmodified.

Burgess' notion is going to have some odd consequences, like if during a chat session the 'z' button on my keyboard suddenly stops working, then it will be helpful to stop using the term 'zipper' in a disagreement over whether Elias Howe invented the zipper. However, it seems odd to say that 'Elias Howe invented the zipper' is constitutive of 'zipper'. There are exceptions in the other direction as well. When we talk to Graham Priest about the paradoxes affecting truth, we know he accepts that some things are both true and not true. That violates a constitutive principle for us—one for 'and' and 'not'. Graham knows at least one of us does not accept T-In (e.g., if p then <p> is true) and T-Out (if <p> is true, then p), which he takes to be constitutive of 'true'. We all keep track of these differences when we have conversations and it does not cause any problems as long as we have not been at the pub too long. So would it be helpful to stop using 'true' or to add 'dialetheic' to Graham's truth claims? Not for us, and we think Graham feels the same way. Does that make T-In and T-Out any less constitutive for 'true'? No. Nevertheless, Burgess' notion of pragmatic analyticity will do for our purposes here.[11]

2. Cases

In this section, we sketch four examples of (what we take to be) inconsistent concepts. Again, those are concepts whose constitutive principles are inconsistent, either with each other or with otherwise uncontroversial facts. Our purpose is to sketch the options for what to do with inconsistent concepts and, in particular, when they are up for replacement. So we need some cases.

We will not argue in detail that these four concepts *are* inconsistent. For one thing, that would take us too far afield. For another, given the vagueness and context sensitivity of 'constitutive

[11] Perhaps Burgess could avoid this problem by appealing to the fact that his notion of analyticity is supposed to be context dependent. Either way, we will not pursue it here.

principle', we realize that there are other interpretive options available, and that there may be contexts in which some of these concepts are not inconsistent. The distraction would not further the present agenda. We ask readers to go along with our classification, at least for the sake of argument.

2.1 Truth

Our primary example, of course, is the concept of truth. There are a number of choices for its constitutive principles, and we need not settle on a single batch of them. The usual candidates are the T-scheme:

$T\langle p\rangle \equiv p$,

or, perhaps, its constituent conditionals:

(T-In) If p, then $T\langle p\rangle$,

(T-Out) If $T\langle p\rangle$ then p.

one instance for each appropriate sentence p. Instead, one might take the introduction and elimination rules:

(T-Intro) from p infer $T\langle p\rangle$

(T-Elim) from $T\langle p\rangle$ infer p;

to be constitutive.[12] In a classical setting, the inference rules stand or fall with the conditionals, but they can come apart in certain non-classical logics. We won't bother reminding readers how these principles lead to contradiction, using more or less uncontroversial rules of inference.

We fully realize that there are many philosophers who do not regard truth as inconsistent (and, indeed, many who reject the very notion of an inconsistent concept). Some argue that truth is

[12] It is common to mention the fact that coreferential singular terms can be substituted in a truth attribution salva veritate, but this principle seems to be constitutive for any predicate and is not specific to truth.

context sensitive, in a way that blocks any derivation of a contradiction[13]; others claim that once we realize what the proper logic is, we will see that no contradiction is forthcoming.[14] Again, we do not engage any of that literature here. The inconsistency of truth is a sort of working hypothesis here. See Scharp (2008, 2013) for an extended argument.

2.2 Set/Membership

As noted by Kurt Gödel (1944, 1964), and a host of others, there are at least two different conceptions of set. According to one of them, a set as a collection of previously given objects. We might speak of sets of US Senators or sets of natural numbers. In this sense, 'set' is a sort of context-sensitive expression, perhaps with an elided constituent.[151] 'Set' would be short for 'set of *X*'s', where context would indicate what goes in for *X*.

As Gödel noted, this conception of set can be iterated. We can speak of sets of sets of senators and sets of sets of natural numbers. And sets of sets of sets of senators, and so on. And, of course, the iteration can be carried into the transfinite. So let us call this the *iterative* conception. The *X*'s, upon which we start the iteration, are sometimes called *urelements*. In some more or less standard interpretations of pure Zermelo Fraenkel set theory, there are no urelements. We begin the iteration on nothing, producing the empty set; and we go on from there. Gödel remarks that the iterative notion of set has not been shown to be inconsistent, nor do the usual arguments affect it, at least not directly. It is captured by the axioms of Zermelo or perhaps Zermelo-Fraenkel set theory, with or without urelements.

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[13] See Glanzberg (2004) for example.
[14] See Field (2008) for example.
[151]Shapiro (1991) misleadingly calls a version of this notion the "logical" conception of set, and used it to interpret second-order languages.

The other notion of set, which we can call the *logical conception*, takes a set to be the extension of a property, concept, propositional function, or predicate. For convenience, we'll stick to properties. The idea is that each logical set is tied to at least one property. The main principle for individuating logical sets is, of course, extensionality: sets are identical just in case their corresponding properties are coextensive. If we add to this a principle that every property has an extension, then we have Gottlob Frege's Basic Law V, and inconsistency.

To wax metaphysical, a logical set is tied to a property, etc., or to a class of properties, while an iterative set is constituted by its members. So the two notions may have different modal profiles, but such issues will not be pursued here. Mathematics is generally taken to be thoroughly extensional.

For purposes of this article, we simply stipulate that the logical notion of set is inconsistent, with Basic Law V, or something similar to that, as a constitutive principle. This might be challenged, but we won't take up such challenges here.

2.3 (Naïve) Infinitesimal

An infinitesimal is a number that is greater than zero but smaller than every positive, finite real number. That is, if e is an infinitesimal, then $e>0$ and for each natural number n, $e<1/n$. One key feature of infinitesimals is that one can divide by them, or use them in cancellation. So if, for two numbers a, b, if $ae=be$, then $a=b$. A second feature is that one can ignore an infinitesimal in addition—and certainly we can ignore the square of an infinitesimal. In those contexts, we think of an infinitesimal as zero.

Here, for example, is a quick derivation of the derivative of the function $f(x) = x^2$ at a value a. Let e be any infinitesimal. Calculate the difference $f(a+e)-f(a)$. With a bit of algebra, this is $2ae+e^2$.

Ignoring the second term, the difference is $2ae$. Divide by the "distance" e, and we get the familiar derivative $2a$.

Before they were supplanted by the method of limits, infinitesimals played an important role in the development of the calculus. This despite the obvious inconsistencies. First, the only number that can be ignored in addition, or even the only number whose square can be ignored in addition, is zero, and, second, one cannot divide by zero. In *The Analyst*, a scathing attack on the new mathematics, George Berkeley famously called infinitesimals "ghosts of departed quantities".[16]

Some have argued that a Robinson-style non-standard analysis vindicates the historical use of infinitesimals, showing that they are consistent all along.[17] Bryson Brown and Graham Priest [2004] suggested a technique called "chunk and permeate" that makes sense of the practice, perhaps without invoking inconsistent constituent principles.[18] We decline to comment on the historical accuracy of this material, just taking it as a working hypothesis that infinitesimals, at least in some contexts, are indeed inconsistent.

2.4 Secretary Liberation

Our final example, due to Charles Chihara (1979), is more of a fanciful toy. As such, however, it is clean, and so will serve our purposes well.

In a certain society, a number of clubs grew in size, and decided to hire secretaries to help manage their corporate affairs. Some of these secretaries took a keen interest in the members and activities of the club they worked for, and applied for membership to that club. A number of these clubs were composed of snobs, and refused membership to their own secretaries. The

[16] Berkeley (1734: section 35).
[17] Robinson (1966).
[18] Brown and Priest (2004).

disenfranchised secretaries got together and formed their own club, the Secretary Liberation Club. They had only one rule written in their bylaws:

> A person is eligible to join the Secretary Liberation Club if and only if he or she works as a secretary for a club and is not eligible to join that club.

The Secretary Liberation Club was an enormous success. As the number of successful, snobby clubs grew, so did the number of disenfranchised secretaries. Eventually, the Secretary Liberation Club got so large that they decided to hire their own secretary, an efficient man named Pat. Pat got to know the ins and outs of the Secretary Liberation Club, and liked what they were doing; and he developed friendships with many of the members. In time, Pat applied for membership in the Secretary Liberation Club (you didn't see that coming, did you?). After pondering things for a few minutes, the membership committee realized that they had a potential problem. They asked if, per chance, Pat worked as secretary for another club, hoping it was a snobby one. Alas, Pat served as secretary only for the Secretary Liberation Club. They then noticed that a member of the Secretary Liberation Club was about to take maternity leave from her job, and encouraged Pat to seek employment with that club—or perhaps with another snobby club. Pat asked if they were somehow displeased with his work, and were hinting that his services were not wanted any more. They quickly reassured him that, indeed, they were most pleased with his service to the Secretary Liberation Club, and were thinking that he might want to take this opportunity to supplement his income. Pat thanked them for their concern, but told them that, as a single parent, he does not want to spend any more time away from his children. Thus the Secretary Liberation Club was thrown into a crisis.

3. Conditions for replacement: when is the cure to be preferred over the disease?

Let C be an inconsistent term or phrase. Define a *replacement* for C to be a term, or a batch of terms, that is, or are, consistent (at least as is known) and which can play at least some of the roles played by *C* in our linguistic and intellectual lives. In this section, we provide some very general guidelines for when an inconsistent term is a candidate for being replaced. We will put the conditions in the form of questions, and will discuss the various criteria in terms of three of our examples: (logical) set, infinitesimal, and the Secretary Liberation Club, in addition to some other, artificial cases we will encounter along the way. We will get to truth in the next section.

3.1 Does *C* figure in a valuable project?

Scharp (2007) introduces a term 'rable' with the following constitutive principles:

'rable' applies to *x* if *x* is a table.

'rable' disapplies to *x* if *x* is a red thing.

Clearly, 'rable' expresses an inconsistent concept in any environment, such as this one, in which there are, or could be, red tables. Prima facie, however, there is no need to forge a replacement for 'rable', since it plays no role in anything interesting or valuable, except perhaps to provide a clean example of an inconsistent term (in which case a consistent replacement would defeat the purpose).

To get fanciful, imagine a linguistic community that has a term for 'rable', but none for 'table' or 'red'. In that case, we might think of 'table' and/or 'red' as replacements.

Pejorative epithets can also be construed as inconsistent. One might take it as constitutive of a given epithet, *A*, that *A* applies to all and only the members of a certain group of people; and one might take it to be a constitutive truth that all *A*'s are *B*, where *B* is some undesirable property (such as being lazy, boorish, cheap, stupid, . . .) The epithet, so construed, is inconsistent if there are, or could be, any *A*'s that are not *B*'s. An epithet may play some role in some "project" or other, say that of feeding the egos of certain other groups of people, but, presumably, this is not a "project" that we—the authors and readers of this volume—take to be valuable. So there is no need to replace an epithet, unless the language does not have any other way to refer to members of the group of people (and there is some need to refer to those members). Better to just not use the epithet.

With our toy example from the last section, the inconsistent phrase is something like 'person eligible to join the Secretary Liberation Club'. The club is valuable to its members, presumably; and we assume there are no moral issues on the downside. The inconsistent phrase is part of the project of administering the affairs of the club. Without it, there is no Secretary Liberation Club. So we take it that this example is part of a valuable project.

Infinitesimals played an important—essential—role in the development of the calculus and real analysis, from about the middle of the seventeenth century until well into the nineteenth century, and beyond if we take physics and engineering into account. So we have a clear case of the inconsistent terms playing a role in a valuable project, an intellectual project in this case.

Set theory is a branch of mathematics, in its own right. That presumably counts as a valuable project on its own. We think it safe to say that, at least nowadays, it is agreed that a legitimate branch of mathematics must be consistent. And, of course, it is no accident that contemporary set theory concerns the iterative conception of set, not the inconsistent logical conception.

We focus here on the role of sets in two other projects. The logical notion played a key role in logicist accounts of mathematics. This is quite explicit in Frege (1893), where what we call "logical sets" are instances of what he called "courses of values". We take it that logicism is, or was, at least prima facie a valuable project, at least for some people.

Our second project is also foundational, but in a somewhat different sense than logicism. Serious issues concerning the coherence, and perhaps consistency, of certain theories occasionally arise in the normal practice of mathematics, and the community needs a way to deal with those. Mark Wilson illustrates the historical development and acceptance of a space-time with an "affine" structure on the temporal slices:

> . . . the acceptance of . . . non-traditional structures poses a delicate problem for philosophy of mathematics, viz., how can the novel structures be brought under the umbrella of *safe* mathematics? Certainly, we rightly feel, after sufficient doodles have been deposited on coffee shop napkins, that we understand the intended structure . . . But it is hard to find a fully satisfactory way that permits a smooth integration of non-standard structures into mathematics . . . We would hope that "any coherent structure we can dream up is worthy of mathematical study . . . " The rub comes when we try to determine whether a proposed structure is "coherent" or not. Raw "intuition" cannot always be trusted; even the great Riemann accepted structures as coherent that later turned out to be impossible. *Existence principles* beyond "it seems okay to me" are needed to decide whether a proposed novel structure is genuinely coherent . . . [L]ate nineteenth century mathematicians recognized that . . . existence principles . . . need to piggyback eventually upon some accepted range of more traditional mathematical structure, such as the ontological frames of arithmetic or Euclidean geometry. In . . . our century, set theory has become the canonical backdrop to which questions of structural existence are referred.[19]

Within the community of professional mathematicians, if not philosophers, a set-theoretic proof of satisfiability resolves any legitimate questions of existence.

Following Penelope Maddy (1997), consider this passage, entitled "Are sets all there is?", from an early chapter of Yiannis Moschovakis (1994) text:

> [Consider] the "identification" of the . . . geometric line . . . with the set . . . of real numbers, via the correspondence which "identifies" each point . . . with its coordinate . . . What is the precise meaning of this "identification"? *Certainly not that points are real numbers*. Men have always had direct geometric intuitions about points which have nothing to do with their coordinates . . . What we mean by the "identification" . . . is that the correspondence . . . gives a **faithful representation** of [the line] in [the real numbers] which allows us to give arithmetic definitions for all the useful geometric notions and to study the mathematical properties of [the line] **as if points were real numbers** . . . [W]e . . . discover within the universe of sets *faithful representations* of all the mathematical objects we need, and we will study set theory . . . **as if all mathematical objects were sets**. The delicate problem in specific cases is to formulate precisely the correct definition of "faithful representation" and to prove that one such exists.[20]

Maddy articulates mathematical benefits of this sort of foundation:

> The force of set-theoretic foundations is to bring (surrogates for) all mathematical objects and (instantiations of) all mathematical structures into one arena—the universe of sets—which allows the relations and interactions between them to be clearly displayed and investigated . . . [P]erhaps most fundamentally, this single, unified arena for mathematics provides a court of final appeal for questions of mathematical existence and proof: if you

[19] Wilson (1993: 208-209)
[20] Moschovakis (1994: 33-34).

> want to know if there is a mathematical object of a certain sort, you ask (ultimately) if there is a set-theoretic surrogate of that sort; if you want to know if a given statement is provable or disprovable, you mean (ultimately), from the axioms of the theory of sets.
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> . . . vague structures are made more precise, old theorems are given new proofs and unified with other theorems that previously seemed distinct, similar hypotheses are traced at the basis of disparate mathematical fields, existence questions are given explicit meaning, unprovable conjectures can be identified, new hypotheses can settle old open questions, and so on.[21]

The claims that Maddy makes on behalf of set theory can be disputed, but we will take them for granted here. Providing this sort of mathematical foundation is indeed a valuable project.

3.2 Does the inconsistency of *C* inhibit the pursuit of the project?

We can break this question down into two. First, does the inconsistency inhibit the pursuit of the project *actually*? In other words, are there any situations which actually come up in the pursuit of the project—situations that cannot be simply ignored—which cannot be handled due to the inconsistency of *C*? Of course, a positive answer to this would at least strongly suggest that those engaging in the project look to replace the inconsistent term.

A second question is: Does the inconsistency inhibit the project *potentially*? Are there some situations which *could* come up in the pursuit of the project, and, if they were to come up, they could not be handled due to the inconsistency of *C*? We might also ask if the situations are foreseeable, and if they are at all likely to arise.

Once these two questions are sorted out, the situation with the Secretary Liberation Club is pretty straightforward. When their secretary, Pat, applies for membership, then the inconsistency in their bylaws becomes pressing. The committee cannot accept Pat for membership without thereby making him ineligible, nor can they deny him membership, because that would make him eligible.

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[21] Maddy (1997: 26, 34f).

Nor could they decide to not act on the application, since *that* decision thereby renders Pat ineligible—he would not be allowed to join, and so he would be eligible. We suppose the committee could just keep on discussing the application, without acting one way or the other, but that would interfere with the purpose of the club. They would spend all their time discussing the application and could not do anything else. In this case, it seems, they should look around for a replacement—a new and better rule for eligibility.

But now let us go back to the time just after the club was formed. Suppose that, at that time, the club was so small that it would not occur to any member that they may need to hire a secretary one day. Still, one of the founders might notice the potential inconsistency—she may be a professional philosopher, for example, and enjoy pondering all sorts of wild scenarios. Even so, it seems to us that there is no strong pressure to revise the bylaws, replacing the eligibility requirement with a consistent one. The founders might find the scenario—that they will grow large enough to need a secretary, hire one that does not work for a snobby club, and have that person apply for membership—simply too far-fetched to worry about. The philosopher among the founders might be told that they will cross that bridge if they ever come to it. The situation is essentially the same even after they hire Pat, especially if it is far-fetched to think that Pat will ever want to join the club. Why would he want to? Our conclusion is that there is only trouble—pressure to revise the bylaws—when Pat does apply for membership or perhaps when it becomes clear that he is going to apply.

We do not wish to get embroiled in delicate matters concerning the history of infinitesimals, and their eventual replacement in modern analysis. It is a long and complex story. We will rely on the sketch in Kitcher (1983, Chapter 10).[22] It seems that logical critiques of infinitesimals in the *Analyst*

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[22] Even if Kitcher's account is completely inaccurate, we suggest that things could have evolved in that way. That is enough for present purposes. See also Wilson (1993).

did have some impact in England, among followers of Newton, but it had virtually none on mathematicians in the Continent. Mathematicians felt little or no need to focus their talents on the task of finding a replacement. They felt they had better things to do. For about two centuries, mathematicians were able to follow their intuitions, and get many interesting and valuable results within the infinitesimal calculus. Moreover, these results displayed a strong mutual coherence with each other, and most could be verified using more traditional methods. Suppose, for example, that someone proved, using infinitesimals, that a certain sequence converges to $\pi/4$. They were then able to verify the accuracy of the result to any desired approximation by calculating members of the sequence.

It is not clear at all clear that anyone could give a batch of rules that exactly codified the practice of using infinitesimals. The practitioners just seemed to know what to do and, more important, what not to do with them. They developed fine intuitions concerning what could be done with infinitesimals and what could not be done, and were able to impart those intuitions to their students.

This started to change sometime during the nineteenth century, when some important results came in conflict with other results. In retrospect—after the calculus was put on a rigorous foundation—we can see what went wrong. For example, one result would be correct (by contemporary lights) if 'convergence' meant *pointwise convergence*, and another, seemingly conflicting result required convergence to be uniform convergence. According to Kitcher, it was situations like this, internal to the normal practice of mathematics, that motivated the pioneering work to make analysis more "rigorous", giving birth to the ε-δ definitions we use today—and to the banishing of infinitesimals, at least for a while.

The lesson of this episode (at least as we have characterized it) is that inconsistent terms need not undermine an otherwise successful and productive intellectual project. The project can go on so long as the practitioners have a good feel for what they can and cannot do with the potentially troublesome terms. And this particular project went on splendidly for some 200 or 300 years, engaging some of the finest mathematical minds ever. Until the trouble arose, internally, the project was not regarded as broken, and was in no need of fixing.

To take one more batch of examples, some philosophers argue that all vague words express inconsistent concepts in the foregoing sense.[23] The idea, it seems, is that a tolerance principle—something like the main premise of a sorites argument—is constitutive of the term in question. Even if we assume that such views are correct, the inconsistency does not prevent vague terms from being applied in everyday life. Ordinary speakers just seem to know, tacitly, not to run too far down a sorites series.

One crucial difference between the mathematical cases and the ordinary ones, concerning vagueness, and perhaps the Secretary Liberation Club, is that mathematics (and science, to the extent that it relies on mathematics) thrives on long chains of deductive reasoning. So inconsistencies are more dangerous. If a mathematical concept is inconsistent, it seems quite likely that, eventually, in the course of a long argument, or several interlocking long arguments, the practitioners will lose track of the adjustments required to avoid trouble, or else their intuitions for what they can and cannot do will eventually fail. Nevertheless, the enterprise can get a good run with inconsistent concepts.[24]

In the previous section, we noted two projects involving the notion of set: Fregean logicism and mathematical foundation. Scholars differ on what the goals of logicism were. One of Frege's stated

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[23] Eklund (2005), Tappenden (1993), and Kamp (1981).

[24] The epigraph of Wilson (1994) is a quip from Oliver Heaviside: "Shall a man refuse his dinner if he does not understand the process of digestion". Another, related Heaviside quip is, "Logic can be patient, for it is eternal".

was to establish that arithmetic is analytic—that its truths could be derived from basic logical laws and definitions. Of course, it turned out that one of those supposed basic logical laws, the fifth, is inconsistent. Frege, at least, took the news of this inconsistency as undermining the program, writing to Bertrand Russell, "Your discovery of the contradiction caused me the greatest surprise and, I would almost say, consternation, since it has shaken the basis on which I intended to build arithmetic."[25]

It seems clear that an inconsistent term cannot play a role in a Fregean logicism, at least not if the target theory, arithmetic in this case, is based on classical logic. One of the stated goals of Frege's logic was to provide a framework for derivations that are absolutely valid and free from gaps. The purpose of that is to establish that the theorems of arithmetic do not invoke anything like Kantian intuition. We cannot rely on tacit rules of thumb or our instincts concerning what can and cannot be done, for if we do, something like Kantian intuition may have snuck in. To serve Frege's purposes, the rules of derivation must be explicitly stated, in full generality. If any sentence at all can be derived in the system, it is useless.

It might be added that Frege took himself to be continuing the program of making analysis rigorous, the same program that led to the banishing of infinitesimals. Inconsistent concepts have no role to play.

Consider next the role of sets in the mathematical foundational program sketched above. According to Maddy (1997: 26), set theory provides "a court of final appeal for questions of mathematical existence and proof: if you want to know if there is a mathematical object of a certain sort, you ask (ultimately) if there is a set-theoretic surrogate of that sort; if you want to know if a given statement is provable or disprovable, you mean (ultimately), from the axioms of the theory of sets". If it is to be *final* appeal, it has to be consistent. When adjudicating matters in this sense, we

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[25] van Heijenoort (1967: 126-128)

cannot rely on intuition or instinct. The rules of inference must be explicit, and fully general. It would seem that an inconsistent term cannot figure in a foundation like this. And, of course, the set theorist does not invoke the *logical* notion of set for this purpose.

3.3 Can the trouble brought by *C* be avoided?

We can break this down into two questions as well. Can the trouble be avoided in every case whatsoever? Can the trouble be avoided in cases that we care about, in the normal pursuit of the project in question. If so, then perhaps the other cases can be safely ignored.

In the case of the Secretary Liberation Club, there is only one case that makes trouble: Pat's application for membership. As the scenario is written, the members of the club do care about that case. The trouble *can* be avoided without revising the rules for membership. In describing the club, we had the membership committee encourage Pat to work at a snobby club, which would make him eligible to join their own club. When that didn't work, they could solve, or at least postpone, their problem by firing Pat. They could query future applicants for the position on whether they might want to join the club. Or they could decide to do without a secretary. This may be a case where the solution is worse than the problem. Presumably, they are happy with Pat's work, and like him. And they do need a secretary.

Here is a fanciful way to avoid the problem and keep Pat on board as their secretary. The directors of the club could charter another club, which they might call the SuperSnob Club. It has a simple rule in its bylaws: no one at all is eligible to join the SuperSnob Club. Then they could add a clause to their own bylaws, stating that anyone who serves as secretary for the Secretary Liberation Club is thereby also secretary to the SuperSnob Club. So Pat, their beloved secretary, is thereby also secretary of the SuperSnob Club. Moreover, he is ineligible to join the SuperSnob Club—because everyone is ineligible to join that club. So, after all, Pat *is* eligible to join the Secretary Liberation

Club. And it involved no extra duties on Pat's part. Since the SuperSnob Club has (of necessity) no members, its affairs should be easy to manage.

Turning to our mathematical examples, once the trouble with infinitesimals arose in practice, the mathematical community did not know how to avoid the problems, at least according to the histories we rely on here. We suppose the mathematicians at the time could just lose interest in the problematic cases, but that would be to give up on the project. It is not in the spirit of mathematics to ignore results we don't like. Maybe there was a clever way to proceed, keeping infinitesimals in place, and adjusting elsewhere, but nobody seemed to have found it, at least not then (but see below).

Concerning logicism, Frege, at least, did not know how to avoid the problem. He tried an alternate definition, a small change to Basic Law V. There is no need to decide whether the proposal invoked a replacement notion, since it, too, proved to be inconsistent.

It is interesting that Russell did not find the paradox as troubling to logicism as Frege did. He thought that Frege's definitions of the individual natural numbers to be essentially correct. The paradox is avoided by invoking the language of ramified type theory.[26] The notion employed is, arguably, a version of the *logical* notion of set, since a set is still thought of as the extension of a predicate (or an attribute). Consistency is maintained by carefully delimiting what counts as a property.[27]

Consider now the mathematical foundational program sketched above. Suppose that someone tried to develop this using the logical notion of set. The inconsistency would undermine the program, for the foregoing reasons. One could try to run the program with the notion of set that comes from ramified type theory. But then the theory is too weak to serve the foundational goals—

[26] Whitehead and Russell (1910).
[27] Russell's "no class" theory does away with sets altogether. Certain constructions on attributes are invoked instead.

for much the same reason that Whitehead and Russell's logicism failed its goals. We'd need an axiom of infinity, and something like the an axiom of reducibility to undermine the effects of ramification. In effect, we'd invoke a version of the iterative notion of set.

3.4 Is a replacement, or batch of replacements, for *C* available?

In other words, are there some other, consistent terms that could be used in place of *C* that will make the project work? The replacements should do much of the work that *C* did, or was intended to do, in the original project. And it should be the case that the new project—founded on the replacement(s)—is somehow better for pursuing the goals of the original project. To paraphrase Chihara (1979), once again, the cure should be preferable to the disease.

We can be brief. If the Secretary Liberation Club does not want to avoid the problem in the ways just suggested—by firing Pat or founding an otherwise useless club—they can just amend their own bylaws. Two obvious possibilities are:

> A person is eligible to join the Secretary Liberation Club if and only if he or she works as a secretary for a club other than the Secretary Liberation Club, and is not eligible to join that other club.

> A person is eligible to join the Secretary Liberation Club if and only if he or she either works as secretary for the Secretary Liberation Club, or else he or she works as a secretary for another club and is not eligible to join that club.

The first change would make Pat ineligible, and the second would make him eligible.

Infinitesimals were not actually replaced. There is nothing in contemporary (standard) analysis that plays their role. Instead, the mathematical giants developed other ways to define the central mathematical notions: derivative, convergence (uniform and pointwise), continuity (uniform and pointwise), and the like.

To be sure, there are a number of contemporary theories that invoke infinitesimals, such as Abraham Robinson's non-standard analysis and the Kock-Lawvere synthetic differential geometry.[28] One could, we suppose, think of those theories as potential replacements to the traditional infinitesimal calculus. We won't speculate on whether the new infinitesimals are replacements of the old, inconsistent ones.

There is an ongoing research program to develop a sort of neo-Fregean logicism, using something other than the logical notion of set. Scottish neo-logicism begins with abstraction principles of a certain form.[29] It is not much of a stretch to suggest that the number operator used to develop arithmetic in Scottish neo-logicism is a sort of replacement to the logical notion of set.[30]

As noted above, it is conceivable that someone might have tried to develop a mathematical foundation, of the sort described above, using the logical notion of set. When that is shown to be inconsistent, one could look to replace the logical notion within the same program, say with ramified type theory. When *that* proves far too weak and unwieldy, the theorist might consider *replacing* the logical notion with the iterative one.

This completes our sketch of the overall framework. It is time to get down to the central case: truth.

4. Truth

[28] See Robinson (1966) and Kock (2006).
[29] See, for example, Hale and Wright (2001).
[30] The founding principle of the logical notion of set is (something like) Frege's Basic Law V, saying that two concepts have the same extension just in case they are co-extensive. The founding principle of the number operator, now known as Hume's Principle, a statement that two concepts have the same number just in case they are equinumerous.

Like we said, we're going to take it for granted that truth is an inconsistent concept and that its constitutive principles are the instances of the T-schema for an expressively rich language like English. Now we want to consider whether it satisfies the conditions for replacement.

4.1 Truth's projects

Inflationists and deflationists about truth alike agree that truth predicates of natural language serve an important expressive role, or rather, several expressive roles. They can be used to semantically ascend so as to avoid unwanted ontological commitments. For example, philosophers have been worried for a long time about negative existentials like 'Santa Claus does not exist' because they seem to commit one to the existence of whatever is being denied existence. Instead of dealing with this problem head on, one can use the truth predicate and assert "Santa Claus exists' is not true'. All that is purportedly being referred to in this assertion is the sentence 'Santa Claus exists', which certainly exists.

Truth predicates also allow one to indirectly endorse or reject propositions one cannot directly assert or deny. For example, even if one does not know, or forgot, what the Poincare conjecture is, one can assert 'the Poincare conjecture is true' and thereby endorse it. It allows us to simultaneously endorse lots of propositions that might even be inconvenient or even impossible to assert one by one; e.g., the theorems of Peano Arithmetic are true.

A third expressive role is as a device of generalization. Let's say someone thinks that if most physicists believe that the speed of light is invariant, then the speed of light is invariant. Moreover, they claim that if most physicists believe that there is dark matter, then there is dark matter. In fact, the point is perfectly general—it has nothing to do with these two particular claims. We can state the general point by using quantifiers and the truth predicate: if most physicists believe something, then it is true. Without the truth predicate, we would be unable to formulate something that generalizes over all these instances. It should be clear that there is some overlap between the generalizing function and the endorsement function, but they are distinct (in the example about what physicists believe, the truth predicate is not used to endorse anything because it occurs in the consequent of a conditional).

These expressive roles are clearly valuable and figure in all sorts of important projects. Moreover, there is general consensus that truth plays these roles.

At this point, however, inflationists and deflationists about truth part ways—the latter claim that these expressive roles are truth's only legitimate functions. Once they have been specified, we have said all that can be said about the nature of truth.[31] Inflationists, on the other hand, think that we can provide an analysis or reductive explanation of truth and this vindicates its explanatory role.[32]

Truth has been a remarkably popular explanans. Meaning, validity, proof, inquiry, belief, assertion, necessity, knowledge, analyticity, predication, and reference have all been tied to truth via various proposed explanations. If one thinks that truth is a legitimate explanans and that providing some kind of explanation of these other concepts is worthwhile, then all these are legitimate projects involving truth. Let us consider three of them.

Plato might not have been the first to suggest that knowledge is justified true belief, but the analysis that later philosophers found in his writings has been tremendously influential.[33] Even those epistemologists writing in response to Gettier's famous purported counterexamples debate about the role justification in an analysis or explanation of knowledge—the vast majority of them take it for granted that truth is a necessary condition for a belief to count as knowledge.[34]

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[31] See McGee (1993), Field (2004), and Horwich (1998) for formulations of deflationism
[32] See David (1994), Tennant (1997), Kitcher (2002), and Lynch (2009) for various kinds of inflationism.
[33] Plato (1976: 97d-98a)
[34] Gettier (1963). One might think of a knowledge-first view as an exception since it denies that knowledge has any reductive explanation much less one in terms of truth; see Williamson (2000).

Since Tarski, the following is a common reductive explanation of the concept of logical consequence: a sentence f is a logical consequence of a set of sentence G iff in every model in which all the members of G are true, f is true.[35] This account appeals to truth in a model, which is, arguably, distinct from truth. Nevertheless, one might adopt a similar view according to which a sentence f is a logical consequence of a set of sentences G iff necessarily, if all the members of G are true, then f is true. There is, of course, a vast literature on explanations of logical consequence what matters for our purposes is that one popular way of explaining one of our most important concepts appeals to truth.

The final example is the one we use as our primary example in the rest of the paper: meaning. When considering philosophical discussions of meaning, it is common to distinguish between theories of meaning and meaning theories. A theory of meaning offers an analysis or explanation of meaning; one popular family of theories of meaning have in common the claim that the meaning of a sentence is its truth conditions.[36]

A meaning theory specifies, for some particular language, the meanings of each of its sentences (e.g., by specifying the conditions under which they would be true). The connection between theories of meaning and meaning theories is fairly obvious: a meaning theory specifies for the sentences of particular languages whatever a theory of meaning says meanings are (e.g., truth conditions, inferential roles, verification conditions, context change potentials). Very often linguists and philosophers of language propose a fragment of a meaning theory that applies only to certain linguistic expressions of a particular language; this is often called "providing a semantics" for the expressions in question. There are all kinds of theories of meaning and meaning theories based on

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[35] Tarski (1936). See also Etchemendy (1990) and Beall and Restall (2006).
[36] Davidson (1967). See Williams (1999) for an alternative interpretation of Davidson's views on meaning.

them, but truth-conditional versions already mentioned are popular.[37] We take these projects and others in which truth figures as an explanatory device seriously and think they constitute valuable projects.

4.2 Truth's problems

We begin with truth's expressive role. One striking fact about the paradoxes that affect truth is that they almost never interfere with everyday communication.[38] Moreover, even if someone were to utter a paradoxical sentence in conversation, it would probably not even be noticed since the facts needed to ascertain paradoxicality are often not available to conversational participants.[39] As such, it seems to us that truth poses no actual problem for any project that relies only on its expressive role. It does pose some potential problems, but the likelihood that these would occur seems remote. So the paradoxes do not seem to pose a serious problem for typical deflationists.

The real problems posed by truth's inconsistency confront the explanatory projects. If, for example, one tries to provide a truth-conditional meaning theory for an expressively rich language (i.e., one with a truth predicate, intuitive logical connectives[40], and the capacity to refer to its own syntactic features), then one's meaning theory turns out to be inconsistent. It entails that a liar sentence (i.e., a sentence q that is provably equivalent to 'q is not true') is both true and not true. We assume in what follows that this is a serious problem and grounds for rejecting the meaning theory in question (recall, we are presupposing the unacceptability of dialetheism).

Notice that this problem is not limited to truth-conditional semantics. For example, a dynamic semantic theory contrasts with a truth-conditional theory in that the former attributes context

[37] See Heim and Kratzer (1998) and Chierchia and McConnell-Ginet (2000) for examples.
[38] This led Stephen Yablo to quip (in conversation) "Sure it works in practice, but does it work in theory?"
[39] Kripke (1975) makes much of this observation.
[40] Classical logic, intuitionistic logic, and stronger relevance logics (e.g., R) are enough to derive the contradiction.

change potentials to the target sentences.[41] Nevertheless, such a theory, if it provides an adequate account of sentences containing 'true', entails that truth predicates obey T-In and T-Out (alternatively, it has all the relevant T-sentences as theorems). Thus, it too entails a contradiction when paradoxical sentences are within its scope. The problem, therefore, is a general one facing any semantic theory whatsoever as long as it is intended to apply to the kind of fragment in question. Anyone engaged in the project of providing a semantics for an expressively rich fragment of natural language faces the problem of inconsistency from the paradoxes. It has nothing to do with a truth-conditional approach in particular.

On the contemporary scene, however, there is very little interest (independent of the paradoxes) in providing a semantics for a fragment of English containing 'true'. That is, most theorists seek to provide semantics for interesting or confusing linguistic expressions (e.g., quantifiers, modals, evidentials, and definites). The truth predicate, being just a regular adjective, is relatively uninteresting. We do not deny, of course, that there is a tremendous literature on the paradoxes affecting truth and many of those engaged in this literature offer semantics for truth predicates that are designed to avoid the fate of inconsistency. Our point is rather that, aside from the paradoxes, there seems to be little interest among semanticists in the truth predicate. Thus, there is no problem posed by the paradoxes for *actual* semantic projects targeting the independently interesting linguistic expressions of natural language because the truth predicate does not seem to be among the independently interesting linguistic expressions.

However, if we take the *goal* of natural language semantics to be a meaning theory for an entire natural language, then the paradoxes do pose a problem for this potential project. If we want a semantics for an entire natural language, then, sooner or later, we will have to confront the problems posed by the paradoxes. If, however, we might be content with a piece-meal understanding of the

[41] See van Eijck and Visser (2010).

workings of our natural language, then perhaps the paradoxes need not ever cause us any trouble. At least one of us is confident that the former is a valuable project, and moreover, that it is a project many in the field are explicitly or implicitly committed to pursuing even if it is currently out of reach.[42]

4.3 Avoiding truth's problems

If there is some way to work around the problems posed by the paradoxes for what we take to be valuable projects involving truth, then we might avoid having to think about the daunting task of replacing our concept of truth. Most of those involved in the debate about the nature of truth have for decades treated the paradoxes like harmless puzzles that have no conceivable impact on their projects.[43] So the idea that those interested in pursuing the projects described above can just ignore the paradoxes associated with truth has a long history and has been something like the received view. However, in the last decade or so there seems to be a growing consensus that many of the popular approaches to the paradoxes are not neutral with respect to theories of the nature of truth. This attitude has contributed to greater interaction between the two discussions of truth in the analytic tradition.

One reason for thinking that the paradoxes are relatively harmless for projects involving truth was the popularity of what we might call monster-barring approaches to the paradoxes.[44] These approaches claim to find some problem with the purportedly paradoxical sentences, propositions, assertions or other items involved in the paradoxes. For example, one holds that the liar sentence is meaningless and so poses no threat whatsoever. Although some still advocate these kinds of approaches, they have been on the decline for a long time. And for good reason. We have plenty of

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[42] See Scharp (2013).
[43] See Horwich (1998) for an example.
[44] The term 'monster-barring' comes from Lakatos (1976).

theories of meaning, propositions, etc. and none of them provide any kind of independent support for a monster-barring view. As such, we do not find this way of avoiding the problems posed by the paradoxes promising.

Another way to try to sidestep the problems in question might be to stipulate that one's semantic theory need only specify the meanings of sentences that are commonly used in conversation. The problem here is that it is impossible to tell which sentences might turn out to be useful in some conversational context. We mentioned above that the paradoxes almost never have a deleterious impact on everyday conversations. But the point is not that paradoxical sentences never get asserted. Rather, if they do happen to get asserted, the participants in the conversation probably won't know that they are paradoxical and so just continue with the conversation as if nothing was wrong. Thus, this strategy for avoiding the problems posed by the paradoxes does not seem very plausible.

4.4 Replacements for truth

Although very few theorists have explicitly offered replacements for the concept of truth, we can interpret a wide range of purportedly descriptive theories of truth as if they are theories of one or more replacement concepts.[45] For example, any of the theories of truth that deny either T-In or T-Out are going to offer poor descriptions of our current practice of using 'true'. For one, we use truth predicates as devices of endorsement and rejection. Anyone who uses truth as a device of endorsement thinks that T-Out is valid and anyone who uses it as a device of rejection thinks that T-In is valid (in full generality). If these rules are not valid, then truth does not fully play each of these roles. Instead of thinking of a theory of truth that rejects one of these rules as a poor descriptive

[45] See Shapiro (2010) for discussion of descriptive vs. revisionary readings of theories of truth.

theory, we could instead think of it as defining a new concept that might be used to replace the concept of truth. However, replacements like this are not going to do all the work we expect a truth predicate to do because they do not fully satisfy the functions of endorsement and rejection. Whether these kinds of replacements can do enough of this work to be worthwhile is a delicate issue we do not take up here.

Perhaps we could look for multiple replacements instead. One could think of the concepts in the Tarskian hierarchy as replacements for truth. In fact, this is a plausible way to read Tarski's own proposal. However, since none of them obey T-In, they won't do all of truth's work either.

Vann McGee proposes two concepts that he thinks will do the work of truth. One is a vague truthlike concept and the other is a notion of definiteness. McGee proves some nice results, like he can express the theory of the two concepts in one of its object languages without contradiction.[46] Although his truth predicate fails to obey T-In or T-Out, one might be able to use the definite truth predicate for endorsements and rejections. We have no idea how his concepts might be used in something like a truth conditional semantics or any of the other projects described above.

One of us advocates a pair of replacements as well. One concept obeys T-In but not T-Out and the other obeys T-Out but not T-In. Thus one serves as a device of endorsement and the other as a device of rejection. The theory of these concepts can be expressed in its object language without any worry about contradictions and the theory is also compatible with classical logic. Finally, the two replacements can do truth's explanatory work in the projects discussed here—most notably in formal semantics.[47]

Deciding between various replacements is a complicated business best left for some other occasion. Instead, we can conclude by recapping the situation with truth. Deflationists have little

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[46] See McGee (1991).
[47] See Scharp (2013).

reason to replace truth as long as they stick to the claim that truth predicates should not be used as explanans. Those who do think that truth plays an explanatory role in a valuable project like truth conditional semantics might want to replace truth if they think of a semantics for an entire natural language as a worthwhile goal and do not see much hope for monster-barring strategies for avoiding the paradoxes.

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